1 Introduction

The problems that I solve in this paper are all taken from *More Mathematical Morsels* by Ross Honsberger. They are all solvable using very simple mathermatical tools, only induction, high school trig., and a small amount of calculus are needed. But don't be fooled, some of the problems are **very** hard at least for me.

2 Problems

Prob. What is the greatest integer n such that $n+10|n^2+100$.
Sol. Using long division we obtain $\frac{n^2+100}{n+10} = n^2 - 10n + 100 + \frac{-900}{n+10}$. Obviously $n+10$ must divide 900, hence the largest n is 890.
Prob. Prove that $2^n|(n+1)(n+2)\cdots(2n-1)(2n)$. I proved this differently then the proof shown in text. I use induction where as the proof in the text does not.
Sol. Base case. $n = 1$. Obviously $1 \cdot 2 = 2$, so the statement is true.
Suppose true for n, then $(n+1)(n+2)\cdots(2n) = 2^n \cdot k$ for some $k \in \mathbb{Z}$. Hence we have $((n+1)+1)(n+1+2)\cdots(2n+2) = (n+2)(n+3)\cdots(2n)(2n+1)(2n+2) = \frac{(n+1)(n+2)(n+3)\cdots(2n)(2n+1)(2n+2)}{n+1} = \frac{2^n \cdot k(2n+1)2(n+1)}{n+1} = 2^{n+1} \cdot k \cdot (2n+1)$. And so we've proven the statement is true for $n+1$. The result follows by induction over $\mathbb{N}$.